

2015 Math Analysis Blitz

1. If $A = \begin{bmatrix} 2 & 3 \\ 4 & x \end{bmatrix}$ and $B = \begin{bmatrix} y & 1 \\ -1 & -1 \end{bmatrix}$, then $AB = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$. Find $x + y$.
- a. 1 b. 2 c. 3 d. 4 e. none of these
2. Which of the following equations has a graph of a parabola that intersects the y-axis at only one point and the x-axis at only one point? $y =$
- a. $\frac{1}{2}x^2 - 2x$ b. $|x - 4|$ c. $\frac{5}{x}$ d. $x^2 - 4x - 5$ e. none of these
3. The period of the function $y = 3 - 2 \cos\left(\frac{x}{4} + \pi\right)$ is
- a. $\frac{\pi}{2}$ b. 8π c. 4π d. $\frac{11\pi}{2}$ e. none of these
4. Find the product of all of the solutions of $16^{x^2+x+3} = 32^{x^2+x}$
- a. -1 b. 3 c. -12 d. 2 e. none of these
5. Find the value of $\sin\left(\sin^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right)\right)$.
- a. $-\frac{1}{2}$ b. $\frac{1}{2}$ c. $-\frac{\sqrt{3}}{2}$ d. $\frac{\sqrt{3}}{2}$ e. none of these
6. Find the remainder when $f(x) = x^3 + 2x^2 - 3x - 4$ is divided by $x - 5$.
- a. 120 b. 1 c. -64 d. 156 e. none of these
7. A gear with a radius of 2 centimeters is turning at $\frac{\pi}{11}$ radians per second. What is the linear speed at a point on the outer edge of the gear?
- a. $\frac{\pi}{22}$ cm/sec b. $\frac{11\pi}{2}$ cm/sec c. $\frac{2\pi}{11}$ cm/sec d. 22π cm/sec e. none of these

8. The exact value of $\cos\left(\frac{16\pi}{3}\right)$ is

- a. $\frac{\sqrt{3}}{2}$ b. $-\frac{\sqrt{3}}{2}$ c. $-\frac{1}{2}$ d. $\frac{1}{2}$ e. none of these

9. The exact value of $\cot(750^\circ)$ is

- a. $\frac{\sqrt{3}}{3}$ b. $\sqrt{3}$ c. $-\frac{\sqrt{3}}{3}$ d. $-\sqrt{3}$ e. none of these

10. Let $x = 5 \sec \theta$ and assume $0 \leq \theta \leq \frac{\pi}{2}$. Simplify the expression $\frac{\sqrt{x^2-25}}{x}$.

- a. $\cos \theta \tan \theta$ b. $\tan \theta$ c. $25 \sec \theta \tan \theta$ d. $\sin \theta$ e. none of these

11. Find the domain, D , and the range, R , for the function $f(x) = e^{\sin x}$.

- a. $D = [0, \infty), R = \mathbb{R}$ b. $D = \mathbb{R}, R = [0, \infty)$ c. $D = \mathbb{R}, R = \left[\frac{1}{e}, e\right]$
d. $D = [-1, 1], R = \mathbb{R}$ e. none of these

12. If $[f(x)]^2 - f(x^2) = 7$ and $f(2) = 3$, the value of $f(16)$ is

- a. 0 b. 8 c. -3 d. -10 e. none of these

13. Given that $3 + i$ is a root of $f(x) = x^4 - 8x^3 + 14x^2 + 28x - 80$, the sum of the real roots of f is

- a. 2 b. 8 c. -2 d. -8 e. none of these

14. If $i = \sqrt{-1}$, then $(i + 1)^{13}$ is

- a. $-64 - 64i$ b. $12 + 12i$ c. $12 - 12i$ d. $-8 + 8i$ e. none of these

15. The sum of the solutions of $\sqrt{x + \sqrt{x + 4}} = 4$ is

- a. 33 b. -33 c. 21 d. 0 e. none of these

16. Determine the value of A such that the line whose equation is $Ax + y - 5 = 0$ is perpendicular to the line containing the points $(4, -2)$ and $(-1, 5)$.

- a. 1 b. -1 c. $-\frac{5}{7}$ d. $\frac{5}{7}$ e. none of these

17. The value of $\csc 32^\circ$ is equivalent to which of the following?

- a. $\sin\left(\frac{1}{32}^\circ\right)$ b. $1 - \cot^2 32^\circ$ c. $\frac{\cos 32^\circ}{\tan 32^\circ}$ d. $\sec 58^\circ$ e. none of these

18. What is the exact value of $\frac{1}{\log_2 36} + \frac{1}{\log_3 36}$?

- a. $\ln 6$ b. $e^{\frac{1}{36}}$ c. $\frac{1}{2}$ d. -2 e. none of these

19. If $i = \sqrt{-1}$, then $i^{35}(i^{17} - i^{41})$ is

- a. $2i$ b. $1 - i$ c. $-2i$ d. 0 e. none of these

20. The range of $f(x) = 1 + e^{-x}$ is

- a. $(-\infty, \infty)$ b. $(1, \infty)$ c. $(-1, \infty)$ d. $(-\infty, 1)$ e. none of these

21. The expression $2 \sec^2 x - 2 \sec^2 x \sin^2 x - \sin^2 x - \cos^2 x$ is equivalent to which of the following?

- a. $2 \sec^2 x - 1$ b. 1 c. -1 d. $\frac{2}{\sec^2 x}$ e. none of these

22. The average of two positive integers is 66. One of the integers is the square root of the other. What is the difference between the two integers?

- a. 110 b. 23 c. 1 d. 20 e. none of these

23. Solve for x: $3^{1-x} = 5$

- a. $\log_3 5$ b. $\log_3\left(\frac{3}{5}\right)$ c. $\log_5 3 + 1$ d. $\log_3 5 + 1$ e. none of these

24. The inverse of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is

- a. $f^{-1}(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ b. $f^{-1}(x) = \frac{e^x}{\ln(1-x)}$ c. $f^{-1}(x) = \frac{2 \ln(x)}{\ln(x+1)}$
d. $f^{-1}(x) = \frac{1}{2} \ln\left(\frac{x+1}{1-x}\right)$ e. none of these

25. What is the coefficient of the x^4y^2 term in the binomial expansion of $(2x - 3y)^6$?

- a. 864 b. -720 c. -1440 d. 2160 e. none of these

26. For any value α such that $-1 \leq \alpha \leq 1$, what is the value of $\sin^{-1} \alpha + \cos^{-1} \alpha$?

- a. 0 b. π c. $\frac{\pi}{2}$ d. $\frac{\pi}{4}$ e. none of these

27. What is the algebraic expression equivalent to $\tan(\cos^{-1} b)$?

- a. $\sqrt{1-b^2}$ b. $b\sqrt{b^2-1}$ c. $\frac{\sqrt{b^2-1}}{b}$ d. $\frac{\sqrt{1-b^2}}{b}$ e. none of these

28. If $\log_b 3 = 1.585$, $\log_b 5 = 2.322$, and $\log_b 7 = 2.810$, find $\log_b \frac{125}{21}$

- a. 2.571 b. 8.191 c. 5.741 d. -0.71463 e. none of these

29. Find the area of a triangle with sides of length 4 inches, 6 inches, and 8 inches.

- a. $12\sqrt{2}$ in² b. $3\sqrt{15}$ in² c. $\sqrt{45}$ in² d. 12 in² e. none of these

30. If $\log(x^2y^3) = a$ and $\log\left(\frac{x}{y}\right) = b$, then $\log x$ is

- a. $\frac{a-2b}{5}$ b. $a-2b$ c. $\frac{a+3b}{5}$ d. $3b-a$ e. none of these

31. The sum of the first 10,000 odd numbers is

- a. 49,995,000 b. 100,000,000 c. 20,001 d. 99,990,000 e. none of these

32. Find a value of α for which $\sin(\theta + \alpha) = -\cos\left(\theta + \frac{\pi}{3}\right)$

- a. $\frac{\pi}{6}$ b. $\frac{\pi}{3}$ c. $\frac{2\pi}{3}$ d. $\frac{\pi}{2}$ e. none of these

33. If $f(x) = \frac{x}{x-1}$, find $(f \circ f)(x)$, assuming $x \neq 1$.

- a. $x(x-1)$ b. x c. $\frac{x}{x-1}$ d. $1 - \frac{1}{x}$ e. none of these

34. Find the sum of the series

$$\sum_{k=2}^{\infty} \left(\frac{3}{4}\right)^k$$

- a. 3 b. $\frac{4}{3}$ c. $\frac{9}{4}$ d. 2 e. none of these

35. Find the multiplicative inverse of the complex number $2 - 3i$.

- a. $\frac{1}{2} + \frac{1}{3}i$ b. $-2 + 3i$ c. $2 + 3i$ d. $\frac{5}{2} + \frac{5}{3}i$ e. none of these

36. Suppose that θ is a second-quadrant angle and that $\sin \theta = \frac{7}{25}$. find $\sin 2\theta$.

- a. $\frac{14}{25}$ b. $\frac{14}{625}$ c. $\frac{527}{625}$ d. $-\frac{336}{625}$ e. none of these

37. The graph of $y = \frac{-x^3 + 2x + 1}{x^2 + 2x + 1}$ has an oblique asymptote at

- a. $y = -1$ b. $y = -x + 2$ c. $y = -x$ d. $y = -x - 1$ e. none of these

38. The quadratic function that has x-intercepts at $(1, 0)$ and $(4, 0)$ and goes through the point $(2, 4)$ also goes through which of the following points?

- a. $(0, -1)$ b. $(5, -8)$ c. $(3, 5)$ d. $(5, -5)$ e. none of these

39. Solve $x^5e^x + 3x^4e^x = 4x^3e^x$ for x .

- a. $\{-4, 0, 1\}$ b. $\{-1, 0, 4\}$ c. $\{0\}$ d. $(-\infty, \infty)$ e. none of these

40. If $f(x) = 2x^3 + Cx^2 + Dx - 5$, $f(2) = 3$, and $f(-2) = -37$, find the value of $C + D$.

- a. -18 b. 0 c. -1 d. -2 e. none of these

41. Suppose $\pi < \theta < \frac{3\pi}{2}$ and $\cos \theta = -\frac{4}{5}$. Find $\sin \theta$.

- a. $\frac{4}{5}$ b. $\frac{3}{4}$ c. $-\frac{3}{4}$ d. $-\frac{4}{5}$ e. none of these

42. Find the last digit in the number 3^{459} .

- a. 1 b. 3 c. 9 d. 7 e. none of these

43. Find the number of solutions of $\sec x \tan x = 4 \sin x$ in the interval $[0, 2\pi)$.

- a. 2 b. 3 c. 4 d. 6 e. none of these

44. A ball is thrown vertically upwards, and its distance from the ground is given by $s = 104t - 16t^2$, where s is in terms of feet and t in seconds. The maximum height the ball will rise is

- a. 136 feet b. 120 feet c. 169 feet d. 176 feet e. none of these

45. When $3x^{31} - 2x^{16} + 7x - 2$ is divided by $x + 1$, the remainder is

- a. 6 b. -6 c. 0 d. -14 e. none of these